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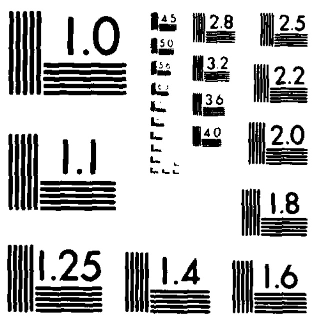

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Aerodynamics Technical Memorandum 320

A SIMPLIFIED DIGITAL FILTER DESIGN PROCEDURE

A.J. FARRELL

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Aerodynamics Technical Memorandum 320

A SIMPLIFIED DIGITAL FILTER DESIGN PROCEDURE.

A.J. FARRELL

SUMMARY

An eight-step procedure for the design of digital Butterworth low-pass filters is outlined which summarises the methods of several standard text-books on digital filter design. Examples of filters designed using the procedure are included.

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CONTENTS

1. INTRODUCTION

In most practical data gathering tasks, the removal of unwanted frequency components from the data by filtering is an essential step. As the cost of microprocessors decreases and as their speed and capabilities increase, digital filters implemented by microprocessors become an attractive alternative to analogue filters, especially in low frequency data gathering applications.

Although there is a great deal of information available on the design of digital filter, it tends to be in a form which is either too limited in scope [1,2] or too tedious or difficult to read for the average reader [3,4,5] .

This memorandum is an attempt to overcome this problem by choosing a commonly used filter configuration, the digital Butterworth low-pass filter, and outlining a step-by-step design procedure drawn from material in the standard text-books on the subject [3,4,5] .

The following sections describe the basic theory and the design procedure; give a worked example and discuss some practical aspects of the working filters.

2. BASIC THEORY

2.1 General

The following sections are a summary of the principles treated in greater detail in references 4 and 5.

In this discussion, it is assumed that original signal of interest is an analogue signal (e.g. the output of an accelerometer) which is sampled at regular intervals. These analogue samples are then digitized by an Analogue-to-Digital Converter (ADC). The data to be digitally filtered are the successive digital words from the ADC output.

For the majority of applications, the unwanted components of an analogue signal are higher in frequency than the desired components, and thus a low pass filter is the required format. Again, a flat amplitude response in the pass-band is more important in most applications than a linear phase response, and so the Butterworth type of filter is chosen.

From the considerations given above, the digital Butterworth low pass filter will fulfil most of the day-to-day filtering requirements and thus has been selected as the design example. However, the extension of the procedures to other types of filter (e.g. chebychev or elliptical) is not difficult.

A digital filter is a digital network which has the property of being frequency selective. It works by multiplying current (in time) and delayed samples of the input data (and in some cases, of the output data) by coefficients and adding the results in an ordered manner. Filters in which the delayed samples of output are used are described as recursive filters and those in which delayed samples of the input are used are known as non-recursive.

An example of a simple recursive digital filter is shown in FIG. 1. The input sample, X(N), is multiplied a coefficient b_o and added to the preceding input sample, X(N-1), which is itself multiplied by a coefficient b_1 to give the intermediate sum:

$$b_o\,X(N) + b_1\,X(N-1)$$

The output is composed of the intermediate sum added to the previous output sample Y(N-1), multiplied by the coefficient, a_1. The output sample Y(N), is thus given by:

$$Y(N) = a_1\,Y(N-1) + b_o\,X(N) + b_1\,X(N-1) \qquad (1)$$

2.2 The Z-Transform

A mathematical transform which has been found to be useful in the representation of digital networks is the Z-transform. The Z-transform, X(Z) of a sequence of N values of X (e.g. the samples of a transducer output) is defined by:

$$X(Z) = \sum_{N=-\infty}^{+\infty} X(N) Z^{-N}$$

where Z is a complex variable. If Z is written in polar form ($Z = re^{jw}$, where w is angular frequency) and if r=1, then the Z-transform:

$$X(Z) = \sum_{N=-\infty}^{\infty} X(N)\,e^{-jwN}$$

is the same as the discrete Fourier transform of the sequence.

The principal merit of using the Z-transform in the synthesis of digital filters is that the Z-transform of a delayed sample X(N-1) is Z^{-1} times the Z-transform of X(N). Thus the Z-transform enables digital delays to be represented mathematically in a convenient way. For example, FIG. 1 could be redrawn with the

unit sample boxes replaced by Z^{-1} boxes (FIG. 2). If FIG. 2 were much more complex with M orders of delay (i.e. the filter order is M) instead of just one (e.g. FIG. 3), the general expression for such a filter would be 4,5 .

$$Y(N) = \sum_{K=1}^{M} a_K Y(N-K) + \sum_{K=0}^{M} b_K X(N-K) \qquad (2)$$

If M=1, we have the situation in FIG. 2.

In the Z-domain, the input and output of a digital filter are related by the system function, H(Z) which is defined as:

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

where Y(Z) is the Z-transform of Y(N) and X(Z) is the Z-transform of X(N).

The filter will in general satisfy the linear, constant-coefficient difference equation

$$\sum_{K=0}^{M} a_K Y(N-K) = \sum_{K=0}^{M} b_K X(N-K)$$

Taking the Z-transform of both sides, and using the properties of the Z-transform, we obtain:

$$Z\left[\sum_{K=0}^{M} a_K Y(N-K)\right] = Z\left[\sum_{K=0}^{M} b_K X(N-K)\right]$$

$$\sum_{K=0}^{M} a_K Z\left[Y(N-K)\right] = \sum_{K=0}^{M} b_K Z\left[X(N-K)\right]$$

$$\sum_{K=0}^{M} a_K Z^{-K} Y(Z) = \sum_{K=0}^{M} b_K Z^{-K} X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{\sum_{K=0}^{M} b_K Z^{-K}}{\sum_{K=0}^{M} a_K Z^{-K}}$$

Thus the definition of a digital filter is a matter of determining the numbers and value of the coefficients b_K and a_K. For example, the system function of a typical sixth order digital filter with a sampling frequency of 60 Hz, a cut-off frequency of 13 Hz, and 0 dB gain the pass band, is:

$$H(Z) = \frac{0,0147(1+6Z^{-1} + 15Z^{-2} + 20Z^{-3} + 15Z^{-4} + 6Z^{-5} + Z^{-6})}{(1-0,8182Z^{-1} + 1,0263Z^{-2} - 0,4243Z^{-3} + 0,1821Z^{-4} - 0,0305Z^{-5} + 0,0032Z^{-6})}$$

and the filter is completely defined by this expression.

2.3 Flow Graphs

The network of FIG. 4 can be more simply represented by a flow graph, as in FIG. 5. Because the filter network is linear, and is also shift-invariant [3] , the order of the two ladder networks in FIG. 4 can be reversed without changing the input/output relationship. Reversing the ladder networks, and eliminating redundant delays, the resulting network [FIG. 5] has the minimum number of delays to realise the general filter and is called a CANONIC form of the filter.

2.4 Filter Design Based on Analogue Design Methods

Many years experience in the design of analogue filters has resulted in well tested and relatively simple design procedures being available. It has been found convenient therefore to design digital filters using methods based on the established analogue practice.

In analogue filter design, the system function which relates the output to the input is normally derived in the S (or Laplace) domain, and is of the general form:

$$H(S) = \frac{Y(S)}{X(S)} = \frac{\sum_{K=0}^{M} d_K S^K}{\sum_{K=0}^{M} C_K S^K} \qquad (4)$$

where Y(S) is the Laplace transform of the output, Y(t) and where X(S) is the Laplace transform of the input, X(t), with t being the time variable, and M being the filter order.

An example of an S-plane system function is the fourth order low-pass filter function with a cut-off frequency of 6 Hz, and unity gain in the pass-band given by:

$$H(S) = \frac{0{,}1994}{S^4 + 1{,}744S^3 + 1{,}5244S^2 + 0{,}7819S + 0{,}2003}$$

The required digital filter system function is the equivalent analogue filter function transformed from the S-plane into the Z-plane.

Several methods have been evolved for the S-plane to Z-plane tranformation. Each of the methods has advantages, but the Bilinear Transformation method has been selected for the following examples because it is simple and straight forward.

2.5 Bilinear Transformation

Equation 3 [Section 2.2] is a differential equation, and the bilinear transformation involves integration of the differential equation, approximation of the resulting integral by the trapezoidal rule, Z-transforming the result, and solving for H(Z) [5]. By inspection of the expressions for H(S) and H(Z), which result from the calculation, it can be seen that the substitution to give the required transformation is:

$$S = \frac{2}{T}\frac{(1-Z^{-1})}{(1+Z^{-1})} \qquad (5)$$

where T is the sample period of the data. Substitutions of this form are generally described in the literature as bilinear transformations.

In the calculations which follow, the data points are assumed for convenience to be a unit time apart. Thus the transformation may be written:

$$S = 2\frac{(1-Z^{-1})}{(1+Z^{-1})} \qquad (6)$$

The effect of this transformation is to map the left hand side of the S-plane into a unit radius circle in the Z-plane (FIG. 6), with the S-plane imaginary axis being transformed into the circumference of the unit circle. The frequency response of the analogue filter is evaluated along the S-plane imaginary ($j\Omega$)

axis, while the frequency response of the digital filter is evaluated along the unit circle, with a maximum range of angle of $\pm\pi$ radians. In practice, this range of $\pm\pi$ radians corresponds to $w_{s/2}$, where w_s is the radian sampling frequency.

A problem associated with this type of transformation is the distorsion in frequency as we go from the S to the Z domain. Along the circumference of the Z-plane unit circle, Z is given by e^{jw}, where w is the radian frequency variable. From equation 6,

$$S = \frac{2(1-e^{-jw})}{(1+e^{-jw})} = Zj\ \tan\left(\frac{W}{2}\right) = j\Omega$$

FIG. 7 is a plot of the digital frequency, w, against the analogue frequency, Ω. It can be seen that the closer to π tnat w becomes, the greater the difference between the analogue and digital frequencies. To overcome this problem, any desired digital filter frequency, w, must be **prewarped**, before the application of the bilinear transformation, to obey the relation:

$$\Omega = Z\ \tan\left(\frac{w}{2}\right) \tag{7}$$

where Ω is the **prewarped** frequency in radians/sec.

2.6 Analogue Butterworth Filters

The general form of the analogue Butterworth filter system function can be written [5] :

$$H(S) = \frac{(\Omega_c)^M}{(S-p_1)(S-p_2)(S-p_3)\dots(S-p_M)} \tag{8}$$

where Ω_c is the cut-off frequency (in radians/second), defined as the frequency at which the system function is 3 decibels (dB) less than the pass-band value and where M is the filter order. The values of p_1 to p_M are the roots of the denominator and are the poles of the system function. When plotted in the S-plane, the poles lie on a circle of radius Ω_c, spaced π/M apart on the circumference, and symmetrically distributed about the imaginary ($j\Omega$) axis [FIG. 8] . Thus to graphically determine the right hand side of equation 8, all that is required is a knowledge of the filter order, M, and the cut-off frequency Ω_c.

In general form, the frequency response of an analogue Butterworth filter is given by [5]

$$|H(j\Omega)| = \sqrt{\frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2M}}}$$

where Ω is any analogue radian frequency, Ω_c is the analogue radian cut-off frequency and M is the filter order. The filter altenuation A in decibels at any frequency is given by the relation:

$$A = 20 \log_{10} |H(j\Omega)|$$

$$= 20 \log_{10} \sqrt{\frac{1}{\left(\frac{\Omega}{\Omega_c}\right)^{2M}}}$$

$$= 10 \log_{10} \left[\frac{1}{(1+\Omega/\Omega_c)^{2M}}\right]$$

Therefore, $10^{A/10} = 1 + (\Omega/\Omega_c)^{2M}$ (9)

Recording equation 9, we obtain:

$$\Omega_c = \Omega\ (10^{A/10} - 1)^{-\frac{1}{2}M} \qquad (10)$$

If our filter requirements are known in terms of the desired altenuations A_1 and A_2 at two respective frequency points, Ω_1 and Ω_2, we can determine the filter order from two simultaneous equations. Eliminating Ω_c, we obtain:

$$M = \frac{\log_{10}\left[(10^{A2/10} - 1)/(10^{A1/10} - 1)\right]}{2 \log_{10} (\Omega_2/\Omega_1)} \qquad (11)$$

3. DESIGN STEPS

The tools required for the design procedure are a pocket calculator, a drawing compass, protractor, linear and log-linear graph paper. The method involves eight steps which are:

(i) Specify the required filter by selecting two critical analogue frequency points, one in the pass band (w_1) and one in the stop-band (w_2)

with the required attenuation at each of these points, (A_1 & A_2).

[N.B.: if the sampling frequency, w_s, is not predetermined, one should be chosen such that it is higher than the stop-band frequency, w_2] .

(ii) Prewarp the critical frequencies (equation 7) to allow for the frequency distorsion inherent in the bilinear transformation.

(iii) Determine the filter order, M, from equation 11. Round up to the nearest integer.

(iv) Determine the cut-off frequency from equation 10.

(v) Graphically determine the analogue system function, H(S). (See section 2.6).

(vi) Apply the bilinear transformation (equation 6) to H(S) to obtain H(Z).

(vii) Draw the flow graph by inspection of H(Z).

(viii) Implement the flow graph as a computer or microprocessor program.

It should be noted that the frequencies w_1 and w_2 in step (i) must in terms of the digital sampling frequency, w_s, which is assigned the value of 2π radians/sec. The filter characteristic is periodic in 2π, with the maximum attenuation being at π radians/sec (the Nyquist or folding frequency). The filter characteristic above π will be a mirror image of the characteristic below π.

4. WORKED EXAMPLE

The design of a 2-pole digital Butterworth low-pass filter is used to illustrate the eight step procedure outlined above.

STEP 1

Assume that the filter requirements are a pass band attenuation of 1 dB at 1,5 Hertz (Hz) and a stop-band attenuation of 15 dB at 6 Hz. Assume a sampling frequency of 60 Hz giving a Nyquist frequency of 30 Hz which is equivalent to π radians/sec. in the digital filter frequency domain. The critical frequencies must be known in terms of the sampling frequency, thus:

$$w_1 = 1,5 \frac{(2\pi)}{60} = 0,1571 \text{ radians/sec.}$$

$$w = 6,0 \frac{(2\pi)}{60} = 0,6283 \text{ radians/sec.}$$

Thus the attenuations $A_1 = 1$ dB at $w_1 = 0,1571$ radians/sec.
and $A_2 = 5$ dB at $w_2 = 0,6283$ radians/sec.

which is the digital filter specification.

STEP 2

Prewarp the critical frequencies

$$\Omega_1 = 2 \tan\left(\frac{w_1}{2}\right) = 0,1574 \text{ radians/sec.}$$

$$\Omega_2 = 2 \tan\left(\frac{w_2}{2}\right) = 0,6498 \text{ radians/sec.}$$

check that $\Omega_1 > w_1$ and $\Omega_2 > w_2$.

STEP 3

Determine the filter order.

$$M = \frac{\log_{10}\left[(10^{0.1}-1)/(10^{1.5}-1)\right]}{2 \log_{10} (0,6498/0,15774)}$$

$$= 1,6831$$

Round M to next hightest integer M = 2.

STEP 4

Determine the cut-off frequency

$$\Omega_c = \Omega_2 \; (10^{1,5} - 1)^{-\frac{1}{2}}$$

$$= 0,2762 \text{ radians/sec.}$$

check that this is close to the requirements.

STEP 5

Determine the S-plane (Laplace) transfer function from the S-plane pole plot for a two-pole analogue Butterworth filter [FIG. 9].

To determine the pole values, draw a circle of radius 0,2762 units and mark the poles at $\pi/2$ apart on the left-hand side of the circle only. The poles must be symmetrically placed about

the imaginary (vertical) axis [see FIG. 8] . The pole positions are then measured from the diagram and the S-plane system function is obtained.

If greater accuracy is required, the pole positions may be determined by calculating the projection of the pole-to-origin distance on the real and imaginary axes since the radius (0,2762) and the angles are known precisely.

The system function (by graphical methods) is:

$$H(S) = \frac{(0{,}2762)^2}{(S+0{,}195+j0{,}195)\ (S+0{,}195-j0{,}195)}$$

$$= \frac{0{,}0763}{S\ +0{,}395+0{,}0761}$$

STEP 6

Apply bilinear transformation.

Substitute $\frac{2(1-Z^{-1})}{1+Z^{-1}}$ for S in the denominator

of the expression for H(S) above

$$S^2 + 0{,}395 + 0{,}0761$$

$$= \frac{4(1-2Z^{-1}+Z^{-2})\ +0{,}78-0{,}78Z^{-2}+0{,}0761+0{,}1522Z^{-1}+0{,}0761Z^{-2}}{1+2Z^{-1}+Z^{-2}}$$

$$= \frac{4{,}8561(1-1{,}6161Z^{-1}+0{,}6788\,Z^{-2})}{1+2Z^{-1}+Z^{-2}}$$

Therefore the Z-plane system function becomes

$$H(Z) = \frac{0{,}0157\ (1+2Z^{-1}+Z^{-2})}{1+1{,}6161Z^{-1}-0{,}6788Z^{-2}}$$

STEP 7

From the system function, draw the flow diagram [FIG. 10] .

STEP 8

Implement the filter as a computer program. FIG. 11 shows the results of implementing the filter on a large digital computer and with a microprocessor. As can be seen from the plot, the objectives, as defined in Step 1 have been achieved.

The further worked examples of digital Butterworth filters are covered in summary in Appendices 1 and 2.

5. DISCUSSION OF RESULTS

In implementing the Z-plane system function, H(Z), care must be taken to ensure the accuracy of the denominator coefficients. For example [FIG. 12] , a 5% error in one of the denominator coefficients of a 4-pole filter produces a complete breakdown of the filter. A 1% error in one denominator coefficient produces a significant error. It is therefore desirable to aim at an accuracy of $\geqslant$0.1% in the denominator coefficients. The denominator coefficient sensitivity can be reduced somewhat by choosing a less efficient structure [3,4] to the canonic form used in the example above.

Related to the problem of coefficient accuracy is the question of rounding errors due to finite register length in microprocessor implementations of digital filters. For many applications, the calculation speed of the most commonly available microprocessors is not sufficient to allow software floating point arithmetic to be used, and so integer arithmetic must be used instead. The errors associated with the finite (8 or 16-bit) register length must be evaluated for each particular filter and the general methods are discussed in references 3 and 4.

An alternative approach is to use one of the special arithmetic chips (e.g. advanced Micro Devices AM9511 or similar) to perform the floating point operations or the integer multiply and divide functions.

A minor point which has been found useful in the microprocessor applications is to scale the coefficients to integer values by multiplying by numbers which are some power of 2 rather than by some power of 10. For example, the coefficient 1,616 could be converted to an integer value by multiplying by 1000. However, a result of sufficient accuracy can be obtained by multiplying by 1024 ($=2^{10}$) which involves a 10-bit shift left rather than a complex multiplication by 1000.

6. CONCLUSION

The majority of practical filtering tasks associated with data gathering can be accomplished using low-pass filters, and this memorandum provides a simplified summary of one of the several text-book approaches to digital filter design. Worked examples are included to illustrate and amplify the summary. The extension of the low-pass methods to cover the high pass and band-pass cases, as well as filter types other than Butterworth, is discussed in a straight forward manner in the referenced texts.

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APPENDIX 1

SUMMARY OF THE DESIGN OF A 4-POLE DIGITAL BUTTERWORTH LOW-PASS FILTER

The sampling frequency is assumed to be 60 Hz, equivalent to 2π radians/sec. in the digital frequency domain. The attenuation requirements are:

-0,5 dB at 4 Hz; -27 dB at 12 Hz.

STEP 1

A_1 = 0,5 dB at w_1 = 0,4189 radians/sec.

A_2 = -27 dB at w_2 = 1,2566 radians/sec.

STEP 2

Ω_1 = 0,4251 radians/sec.

Ω_2 = 1,4530 radians/sec.

STEP 3

M = 3,384

Round to next heightest integer M = 4.

STEP 4

Ω_c = 0,6682 radians/sec.

STEP 5

$$H(S) = \frac{0{,}1994}{S^4+1{,}744S^3+1{,}5244S^2+0{,}7819S+0{,}2003}$$

STEP 6

$$H(Z) = \frac{(0{,}005273)(1+4Z^{-1}+6Z^{-2}+4Z^{-3}+Z^{-4})}{1-2{,}3265Z^{-1}+2{,}2480Z^{-2}-1{,}0161Z^{-3}+0{,}1794Z^{-4}}$$

STEP 7

Flow graph appears in FIG. 13.

STEP 8

Results of computer implementation appear in FIG. 14.

APPENDIX 2

SUMMARY OF THE DESIGN OF A 6-POLE DIGITAL BUTTERWORTH LOW-PASS FILTER

The sampling frequency is assumed to be 60 Hz (= 2π radians/sec. in the digital frequency domain). The attenuation requirements are :

0,2 dB at 9 Hz and 40 dB at 20 Hz.

STEP 1

A_1 = 0,2 dB at w_1 = 0,9425 radians/sec.

A_2 = 40 dB at w_2 = 2,0944 radians/sec.

STEP 2

Ω_1 = 1,0191 radians/sec.

Ω_2 = 3,4641 radians/sec.

STEP 3

M = 5,0122, rounded up to 6.

STEP 4

Ω_c = 1,6079 radians/sec.

STEP 5

$$H(S) = \frac{17,2804}{S^6+6,21S^5+19,2965S^4+38,0149S^3} \text{----}$$

$$\text{------} \frac{}{+49,9034S^2+41,4914S+17,2387}$$

STEP 6

$$H(Z) = \frac{0,014701(1+6Z^{-1}+15Z^{-2}+20Z^{-3}+15Z^{-4}+6Z^{-5}+Z^{-6})}{1-0,8182Z^{-1}+1,0263Z^{-2}-0,4243Z^{-3}+} \text{----}$$

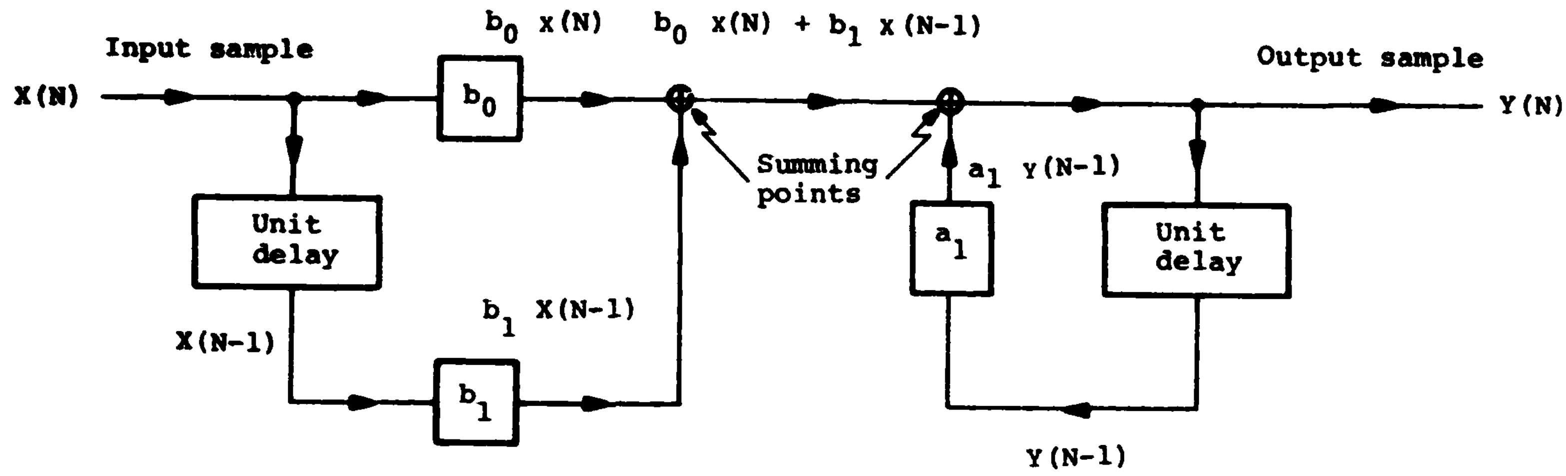
$$\text{----} \frac{}{+0,1821Z^{-4}-0,0305Z^{-5}+0,003211Z^{-6}}$$

STEP 7

Flow graph [FIG. 15] .

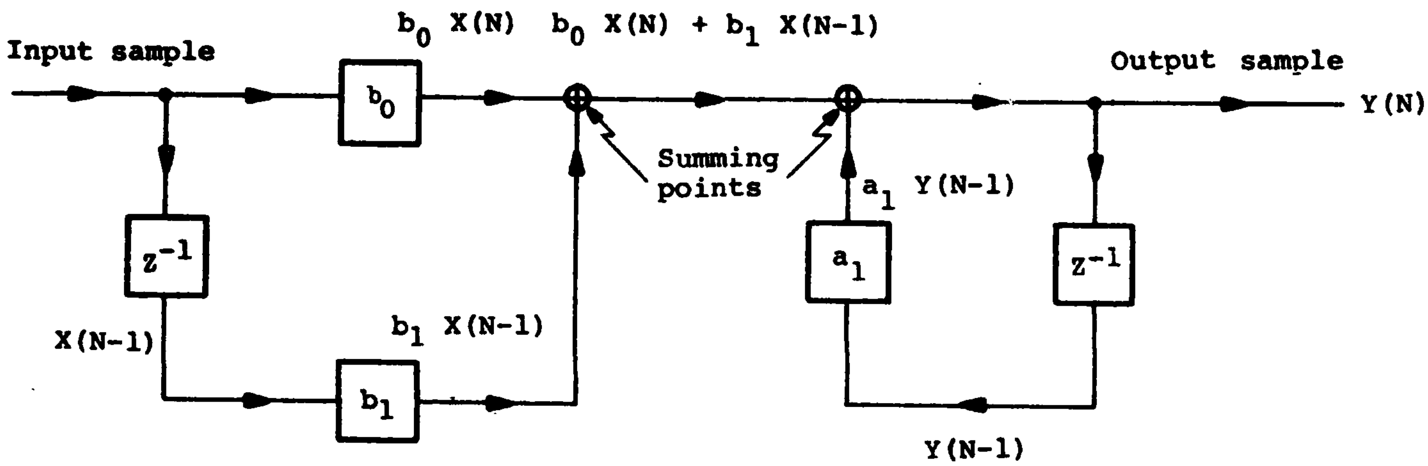
STEP 8

Implement on computer. Frequency characteristic of the resultant filter is shown in FIG. 16.



$$Y(N) = a_1\ Y(N-1) + b_0\ X(N) + b_1\ X(N-1)$$

FIG. 1 SIMPLE RECURSIVE DIGITAL FILTER



$$Y(N) = a_1\ Y(N-1) + b_0\ X(N) + b_1\ X(N-1)$$

FIG. 2 SIMPLE RECURSIVE DIGITAL FILTER WITH z^{-1} INSTEAD OF UNIT DELAY BOXES

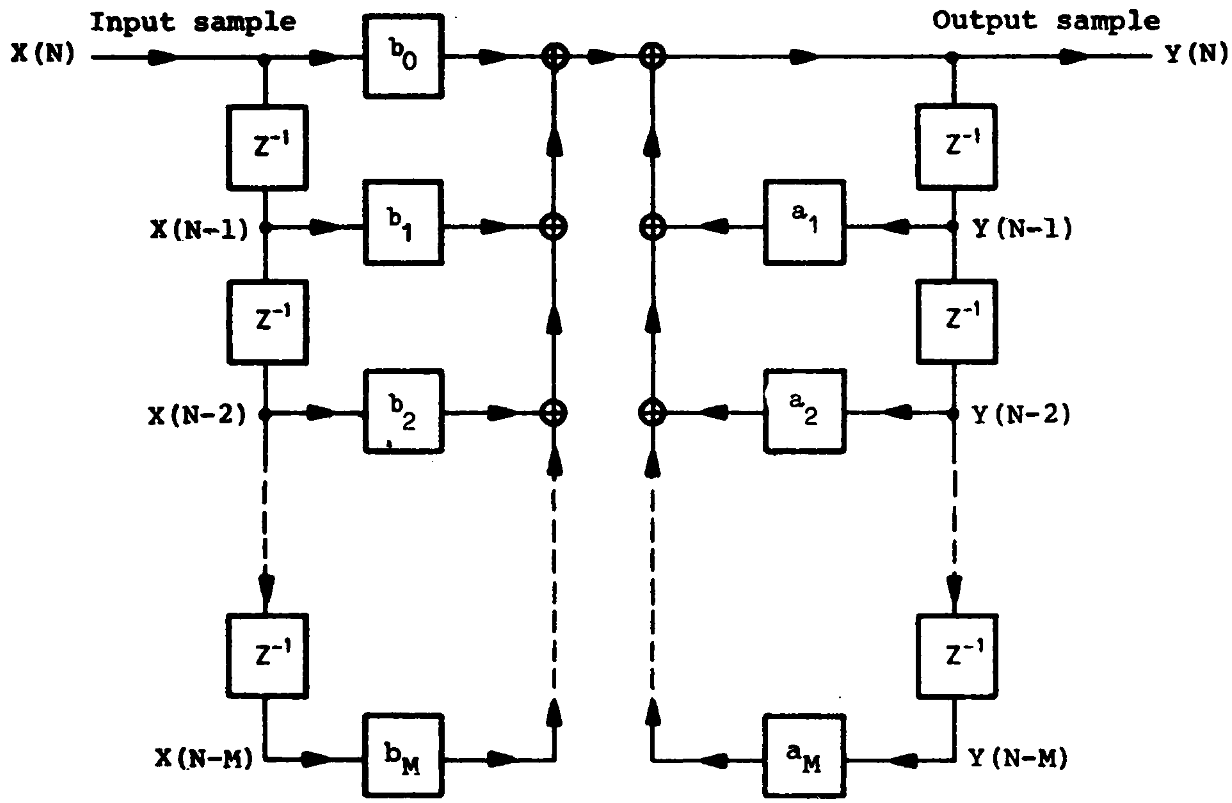


FIG. 3 GENERAL RECURSIVE DITIGAL FILTER OF ORDER M

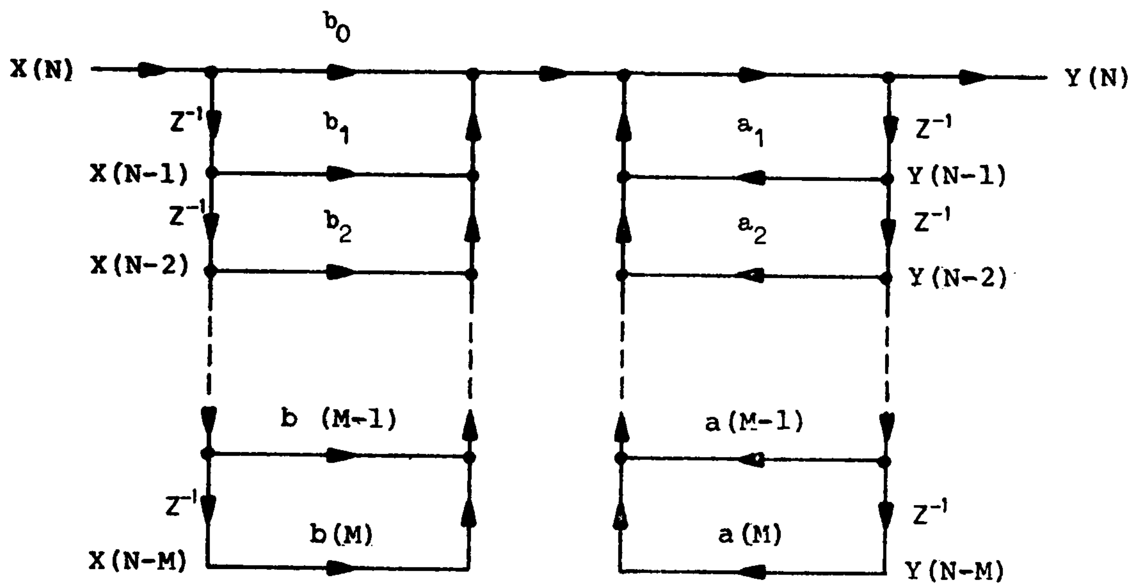


FIG. 4 DIRECT FORM REALIZATION OF GENERAL DIGITAL FILTER OF ORDER M.

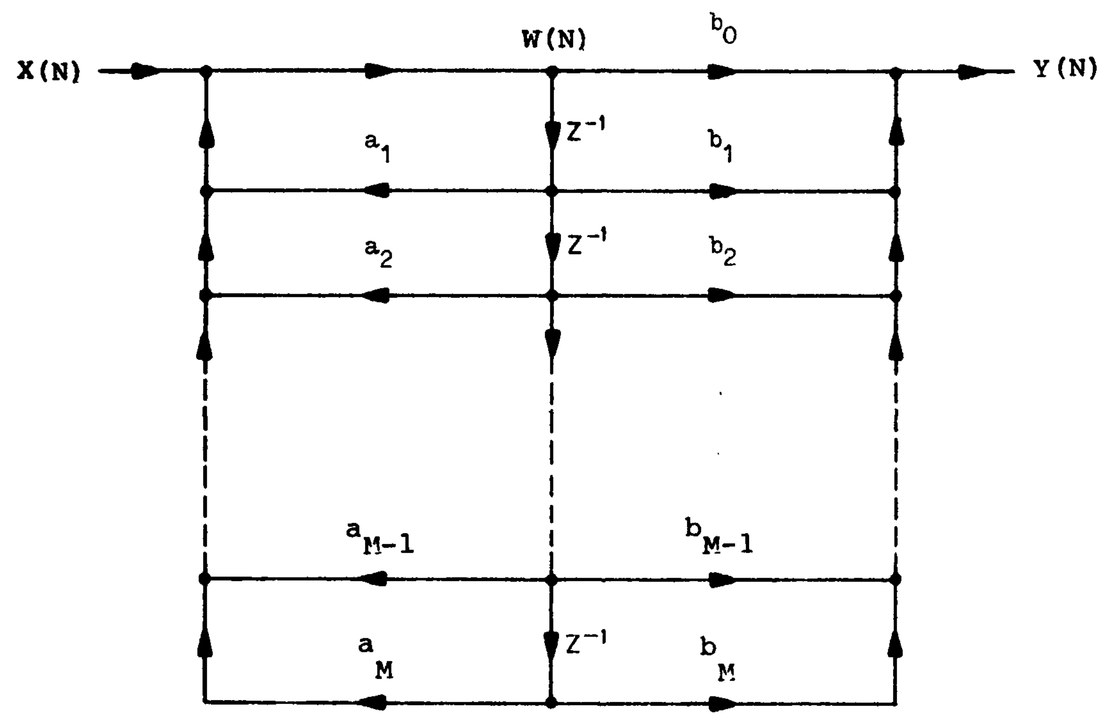


FIG. 5 CANONIC FORM OF GENERAL DIGITAL FILTER OF ORDER M.

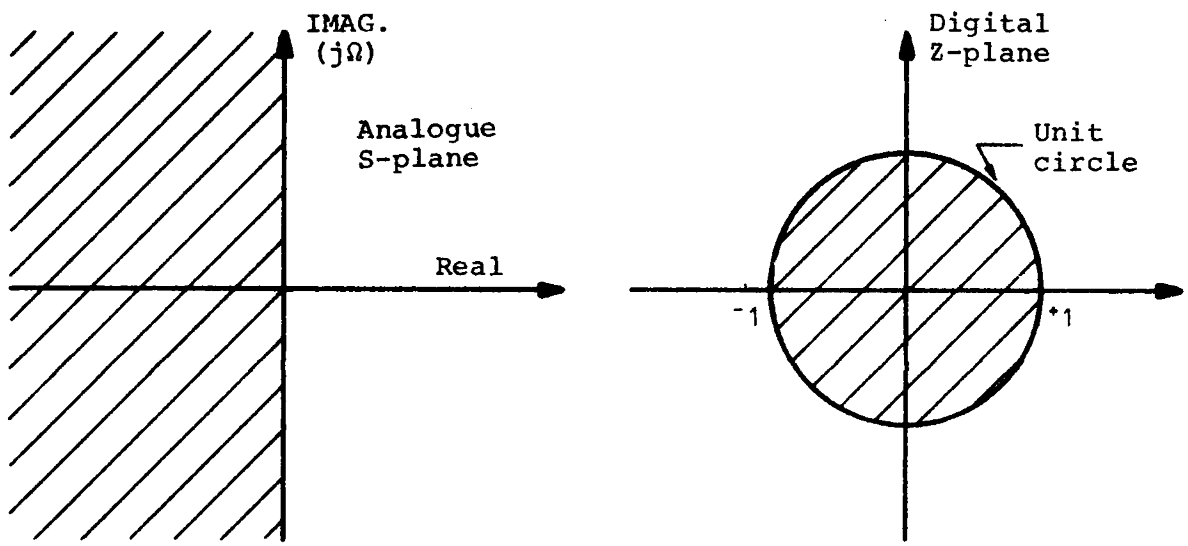


FIG. 6 GRAPHACAL REPRESENTATION OF BILINEAR TRANSFORMATION

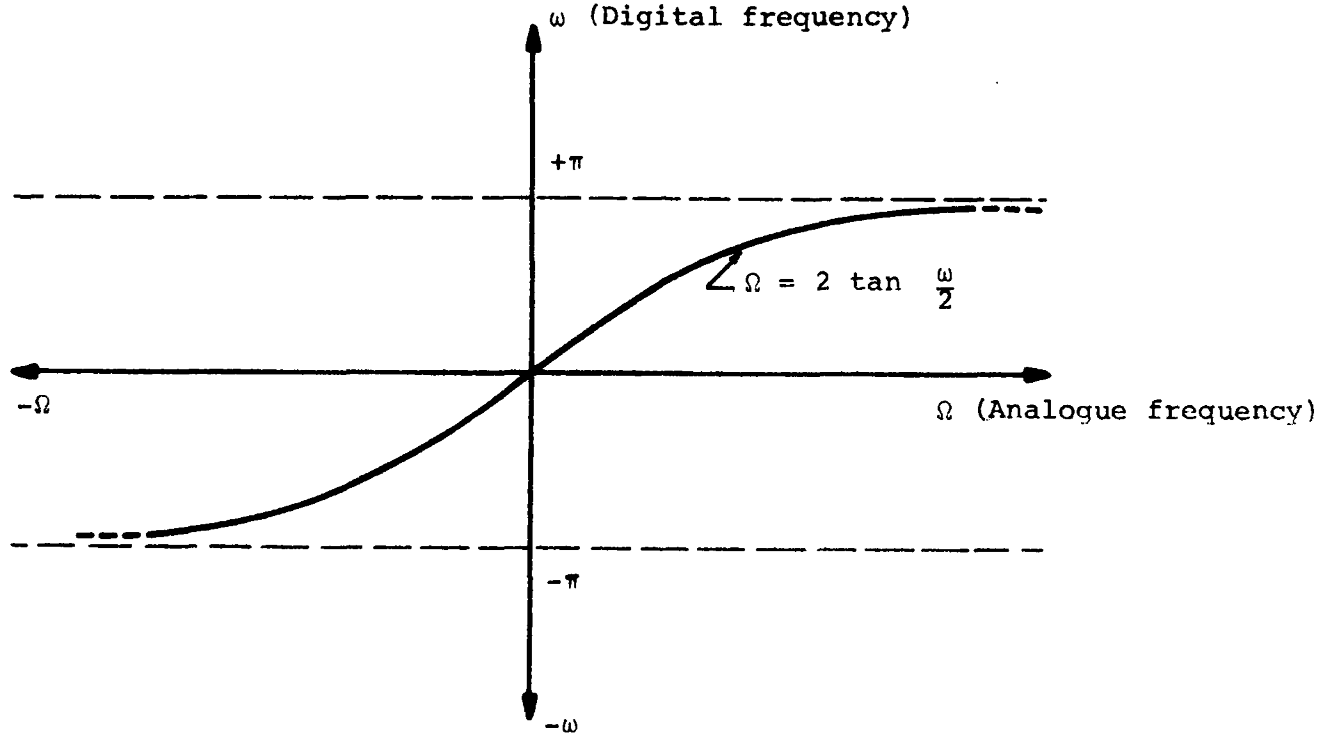


FIG. 7 FREQUENCY DISTORSION DUE TO BILINEAR TRANSFORMATION

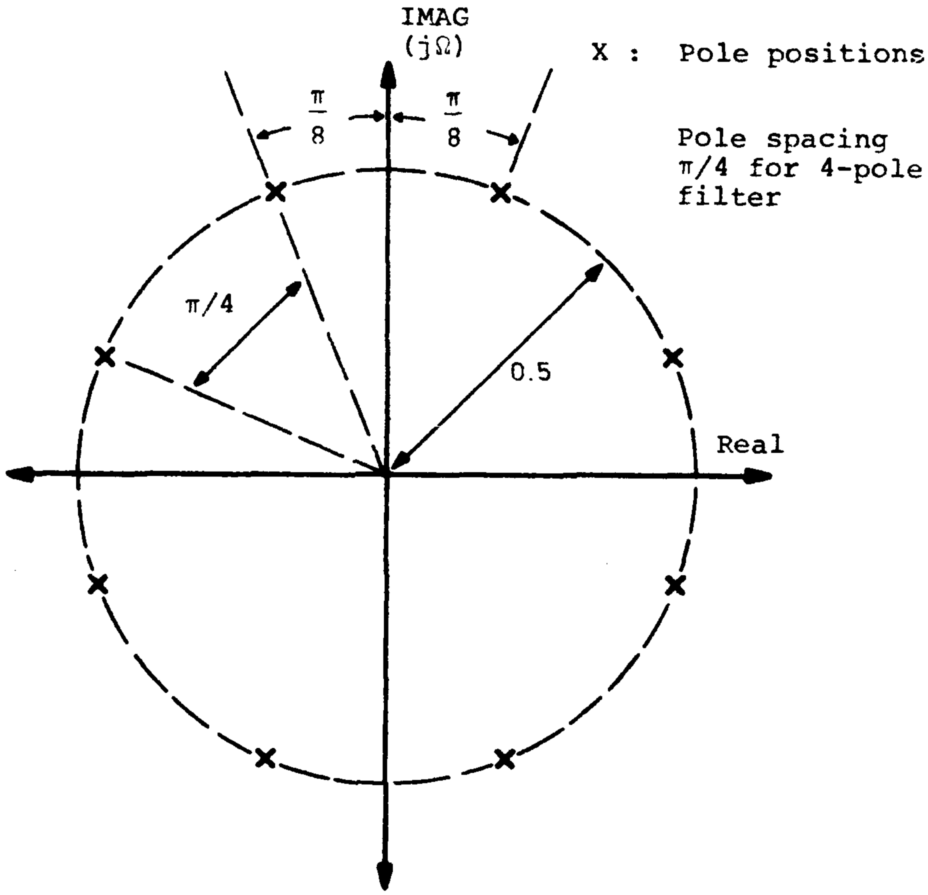


FIG. 8 'BUTTERWORTH CIRCLE', TYPICAL EXAMPLE. POLE PLOT OF 4-POLE BUTTERWORTH ANALOGUE LOW PASS FILTER, Ω_c = 0,5 RADS/SEC.

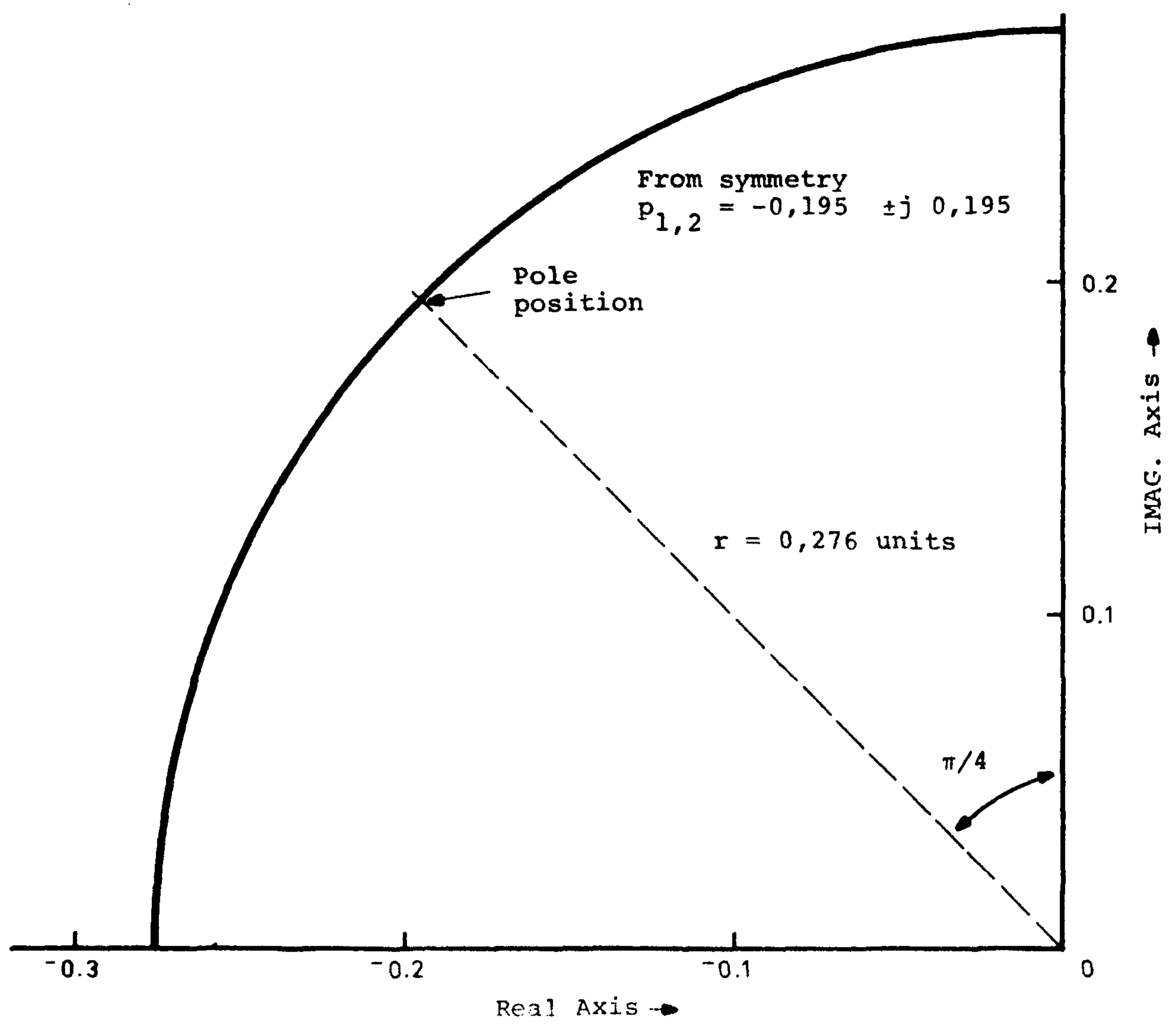
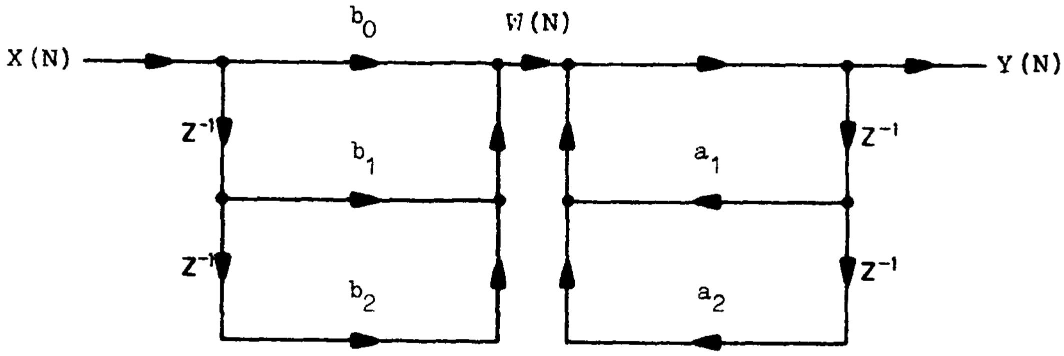


FIG. 9 PART OF BUTTERWORTH CIRCLE [illegible] 2-POLE LOW [illegible] FILTER



$a_1 = 1{,}6161$ $\quad$ $b_0 = 0{,}0157$

$a_2 = -0{,}6788$ $\quad$ $b_1 = 2{,}00$

$b_2 = 1{,}00$

FIG. 10 FLOW DIAGRAM FOR 2-POLE DIGITAL BUTTERWORTH LOW PASS FILTER

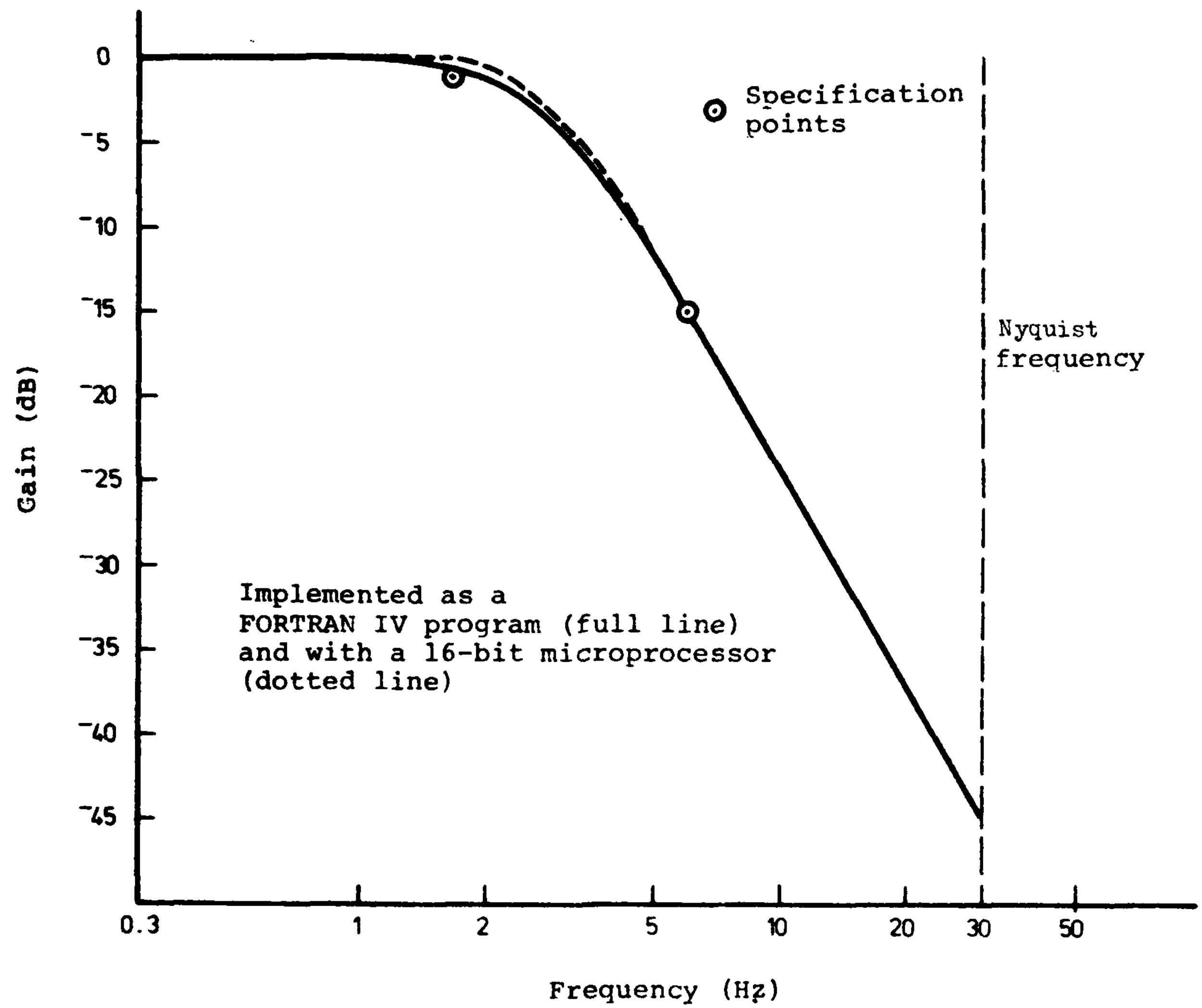


FIG. 11 FREQUENCY CHARACTERISTIC OF 2-POLE DIGITAL FILTER - SAMPLING FREQ: 60 Hz.

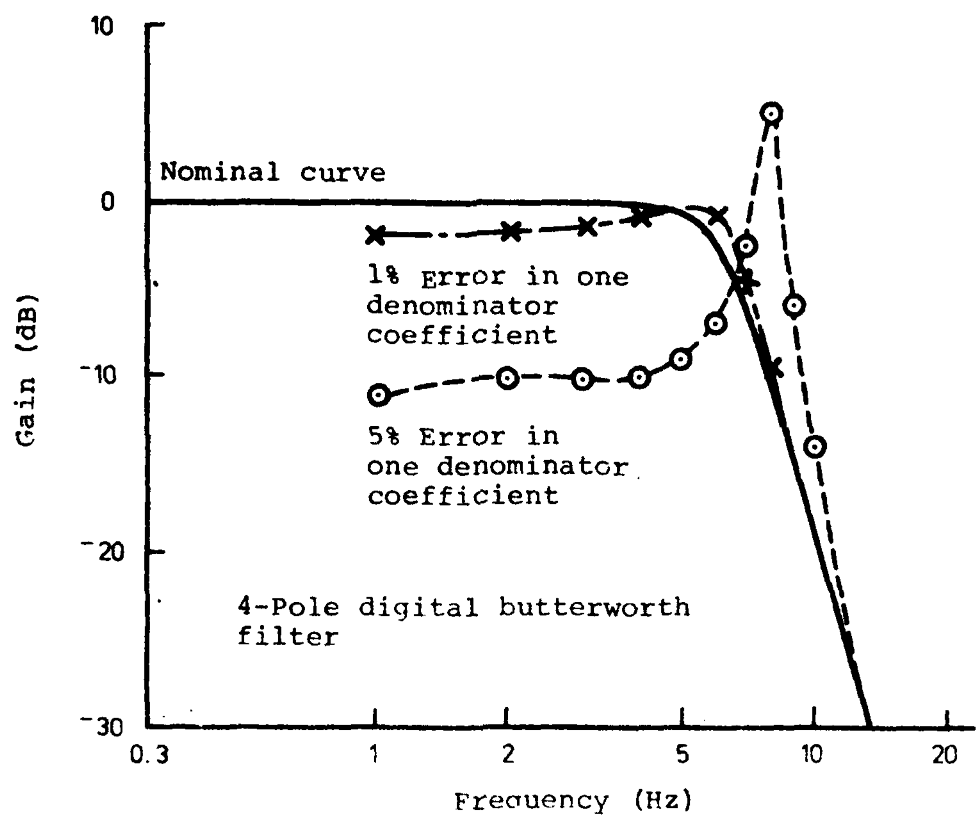
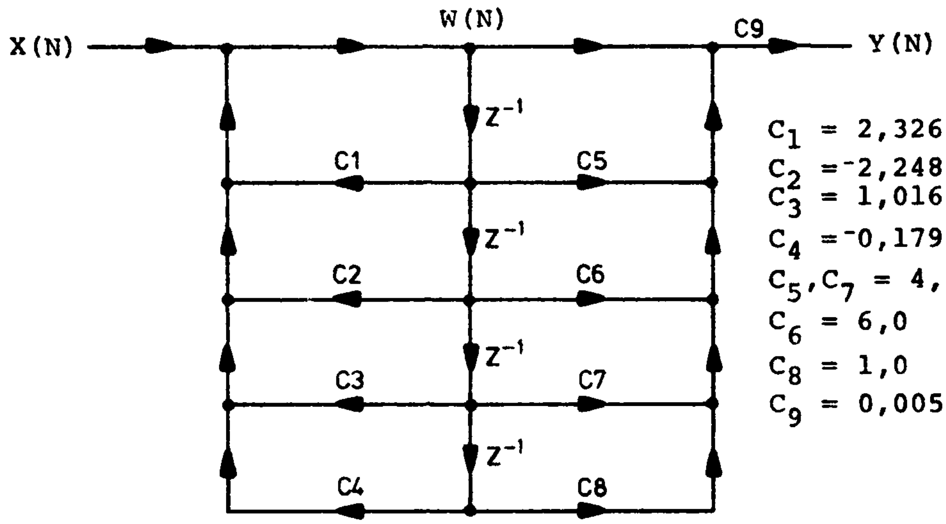

Gain (dB)
10
0
-10
-20
-30
Nominal curve
1% Error in one denominator coefficient
5% Error in one denominator coefficient
4-Pole digital butterworth filter
0.3
1
2
5
10
20
Frequency (Hz)

FIG. 12 EFFEC[illegible]



C_1 = 2,3265
C_2 = ⁻2,2480
C_3 = 1,0161
C_4 = ⁻0,1794
C_5,C_7 = 4,0
C_6 = 6,0
C_8 = 1,0
C_9 = 0,005273

FIG. 13 **FLOW GRAPH OF A 4-POLE DIGITAL BUTTERWORTH LOW-PASS FILTER (DIRECT FORM II (REF. 5))**

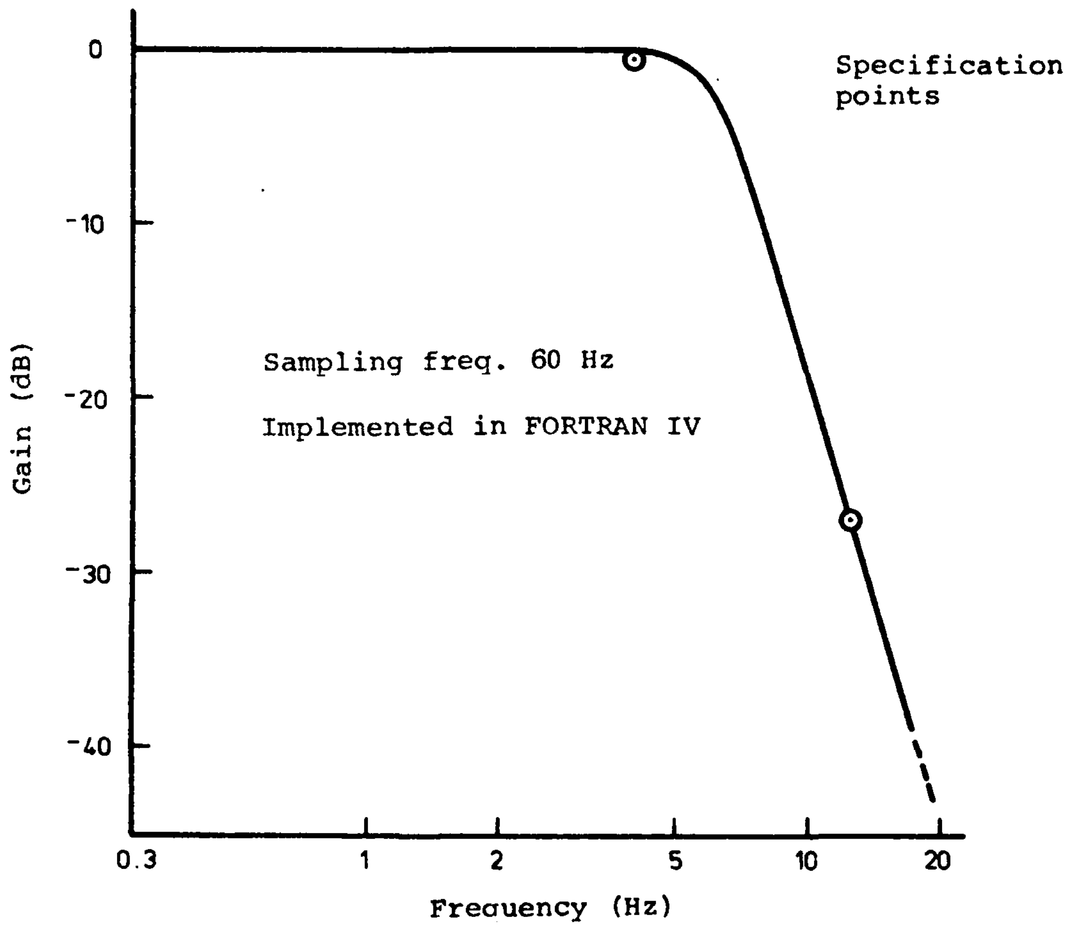


FIG. 14 FREQUENCY CHARACTERISTIC OF 4-POLE DIGITAL BUTTERWORTH FILTER.

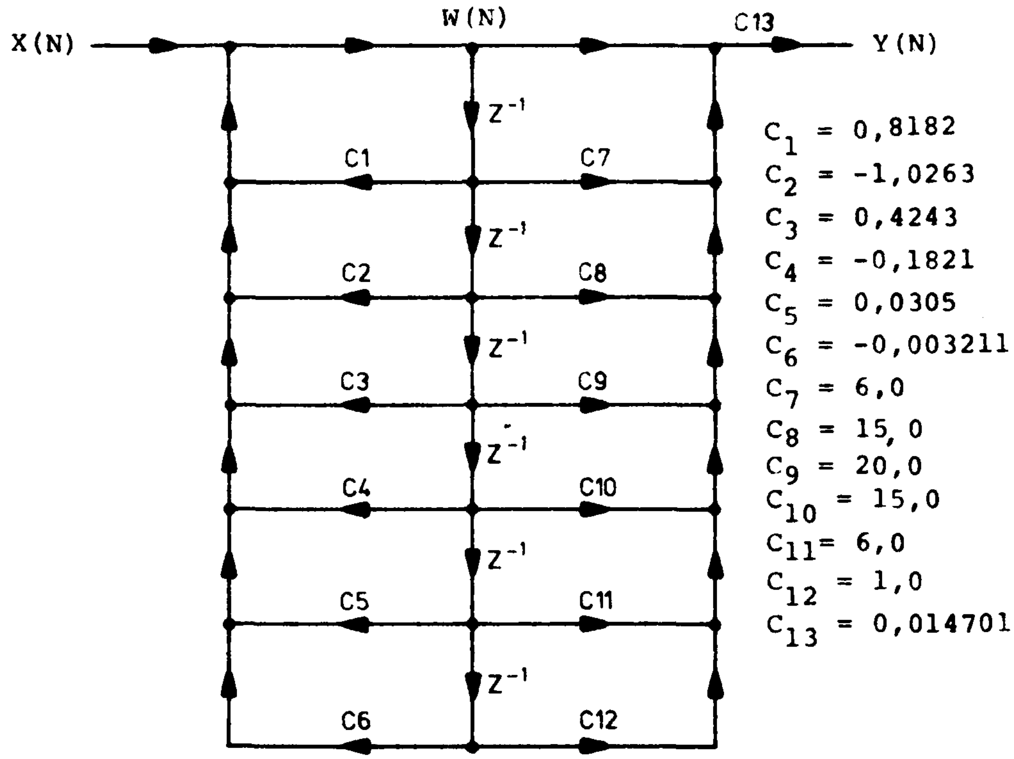


FIG. 15 **FLOW CHART OF 6-POLE LOW PASS DIGITAL BUTTERWORTH FILTER (DIRECT FORM II (REF. 5))**

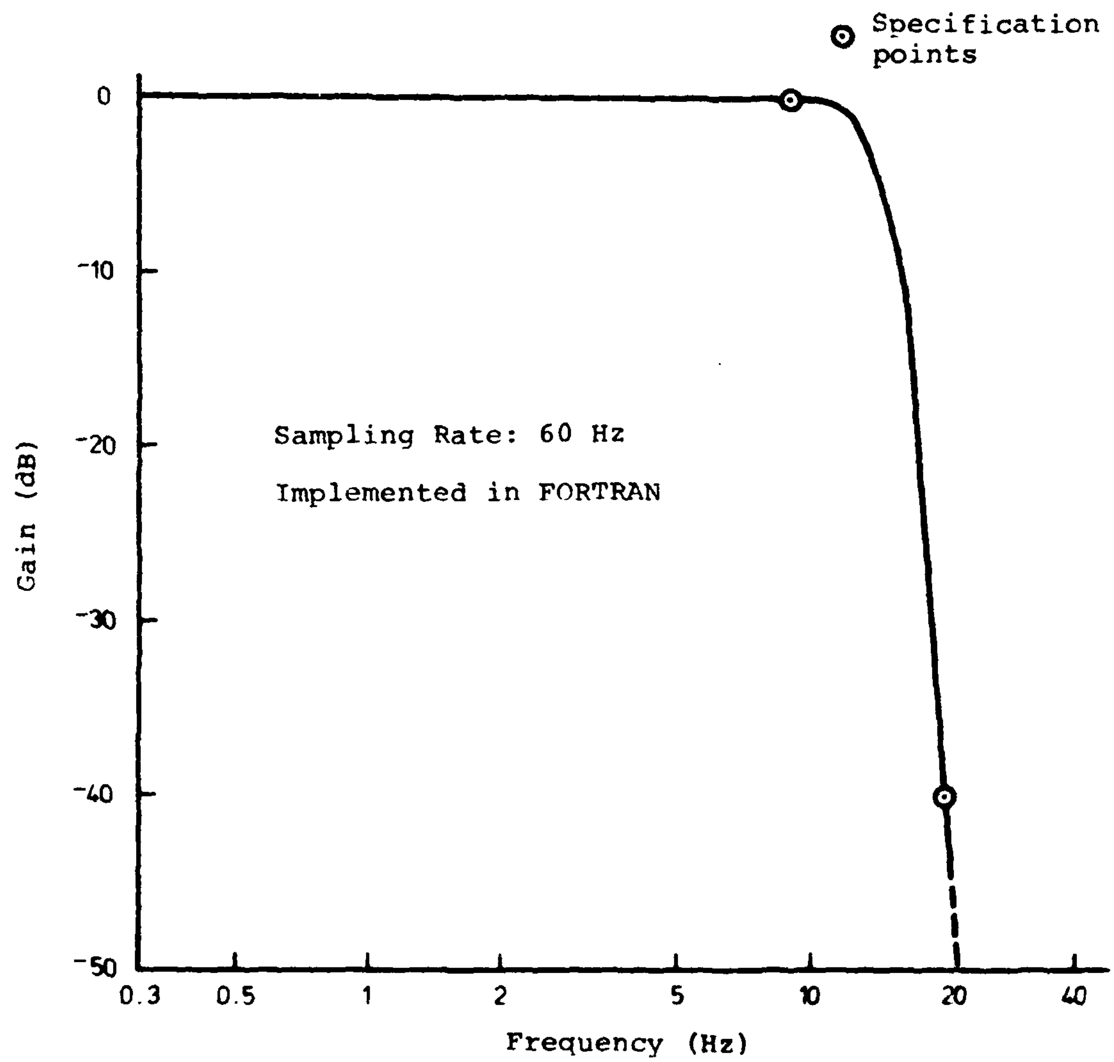


FIG. 16 6-POLE DIGITAL BUTTERWORTH FILTER FREQUENCY CHARACTERISTIC

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